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Some unresolved issues in fluid-structure interactions

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Abstract

Three of the many unresolved or partly resolved issues in fluid-structure interactions are discussed in this paper: (i) the existence of post-divergence flutter of shells with supported ends (an inherently conservative system in the absence of dissipation) subjected to axial flow; (ii) the possible instability of shells with mismatched end-supports containing flow, at infinitesimal flow velocities; (iii) the stability of aspirating cantilevered pipes. The contents of this paper served as the foundation for a plenary lecture delivered at the 8th International Conference on Flow Induced Vibrations (FIV-2004) at Ecole Polytechnique, Paris, in July 2004.

Keywords: Pipes conveying fluid; Stability; Post-divergence flutter; Dynamic divergence; Shells conveying fluid; Aspirating cantilevers; Ocean mining; Feynman's

1. Introduction

The topics to be discussed in this paper came to the surface in the course of writing a book on fluid–structure interactions of slender structures in contact with axial flows (Païdoussis, 2003a). These and other topics have been deliberated upon in a recent conference paper (Païdoussis, 2003b), but naught remains still (*τάπάντα ρεῖ*); thus, resolved issues proved to be not so resolved after all, while significant progress has been made and new insights have come to the fore in others. The contents of this paper are an expanded and updated version of a recent oral presentation (Païdoussis, 2004).

The topics in question are: (i) post-divergence flutter of shells subjected to axial flow; (ii) stability of fluid-conveying shells with different upstream- and downstream-end support conditions; and (iii) stability of aspirating cantilevered pipes, such as those used in ocean mining.

The question of existence of post-divergence flutter of shells containing or immersed in axial flow appears to be resolved at last; although predicted by linear theory, it does not arise in nonlinear theory and it has not been observed in experiments (Karagiozis et al., 2004, 2005). The question is discussed here in a wider context, also including pipes conveying fluid, pulmonary passages with flow, and cylinders in axial flow, and some new insights are gained.

The definitive answer as to whether fluid-conveying shells with the upstream end clamped and the downstream one simply supported actually lose stability at infinitesimal flow velocities has proved to be elusive (Païdoussis, 2003; Zolotarev, 2004). The saga is retold of the see-saw progress towards unravelling this conundrum, and some further thoughts are presented.

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Finally, the question of stability of aspirating cantilevers was thought to have been totally resolved (Païdoussis, 1998, 1999), till it was discovered that the theoretical proof was, at best, incomplete (Païdoussis, 2004; Kuiper and Metrikine, 2005; Païdoussis et al., 2005). This question, which is related to Feynman's quandary on aspirating rotary sprinklers, is re-examined and it is shown that it is only deceptively simple.

2. Post-divergence flutter of slender structures subjected to axial flow

The question of existence of post-divergence flutter of pipes and shells conveying fluid or cylinders in axial flow is posed exclusively for structures with supported ends, which are *inherently conservative systems*; i.e., systems which, if dissipative forces are not taken into account, are conservative.[1] Indeed, cantilevered systems, which are inherently nonconservative, are excluded from this discussion.

2.1. Post-divergence flutter of pipes conveying fluid

We start with a pipe with clamped or simply supported ends conveying fluid; a clamped–clamped system is shown in Fig. 1. The simplest form of the linear equation of motion (for small lateral motions), in the absence of gravity and neglecting dissipation in the material of the pipe and friction with the surrounding ambient fluid, is given by

$$EI\frac{\partial^4 w}{\partial x^4} + M\,U^2\,\frac{\partial^2 w}{\partial x^2} + 2M\,U\,\frac{\partial^2 w}{\partial x \partial t} + (M+m)\frac{\partial^2 w}{\partial t^2} = 0, \tag{1}$$

where EI is the flexural rigidity, $w(x,t)$ the transverse displacement of the pipe, x being the axial coordinate and t the time, U is the flow velocity, M the mass of the fluid per unit length, and m the mass of the pipe per unit length. Hence, the terms in Eq. (1) may be identified as (i) the flexural restoring force, (ii) the centrifugal force associated with radius of curvature $R = (\partial^2 w/\partial x^2)^{-1}$, (iii) the Coriolis force, and (iv) the inertial force. It should be stressed that the absence of viscous-flow terms in Eq. (1) does not signify that the model is inviscid; it is indeed based on a one-dimensional viscous flow model, but the frictional pressure drop and the flow-induced traction (tension) on the pipe cancel each other, exactly in the limit of linear theory (Païdoussis, 1998).

It has been known for a long time — see, e.g., Feodos'ev (1951), Housner (1952) — that, for sufficiently high flow, the pipe loses stability by divergence, i.e., it buckles. This has been confirmed by experiments, by Dodds and Runyan (1965) and others. Then, in the 1970s, Païdoussis and Issid (1974) found that linear theory predicts the existence of post-divergence flutter, at higher flow velocities. A typical Argand diagram for a pinned–pinned (simply supported) system is shown in Fig. 2, where the dimensionless frequency $\omega = \mathscr{Re}(\omega) + \mathrm{i}\,\mathscr{Im}(\omega)$ is plotted parametrically with the dimensionless flow velocity $u = (M/EI)^{1/2}UL$. It is seen that the system loses stability by divergence at $u = \pi$ in its first mode, and at $u = 2\pi$ in its second mode also; subsequently, the first- and second-mode loci coalesce, and coupled-mode flutter is predicted to occur at $u = 6.375$.[2] This finding is problematic, because the work done by the fluid on the pipe in a presumed cycle of oscillation of period T is found from Eq. (1) to be

$$\Delta W = -MU\int_0^T \left[\left(\frac{\partial w}{\partial t}\right)^2 + U\left(\frac{\partial w}{\partial t}\right)\left(\frac{\partial w}{\partial x}\right)\right]\Bigg|_0^L \mathrm{d}t \tag{2}$$

where L the pipe length; hence, since $\partial w/\partial t = 0$ at both $x = 0$ and $x = L$,

$$\Delta W = 0. \tag{3}$$

This, then, constitutes a paradox: for how is it possible for the pipe to flutter if the system is conservative and no energy is supplied to sustain the oscillation? Since viscous effects have been taken into account in deriving the equation of motion, this is a result applicable to both inviscid and (real) viscous flows. In any case, theory predicts coupled-mode flutter, even if dissipation in the pipe material is accounted for.

[1]The classical example of an inherently conservative system is a column with supported ends, subjected to a compressive load. On the other hand, an undamped cantilevered column subjected to a compressive *follower* force (i.e., one remaining tangential to its free end as the column is deflected) is a nonconservative system. The work done on the system by the compressive load is zero in the first case and nonzero in the second; see, e.g., Bolotin (1963), Ziegler (1968). Conservative systems, if subject to instability, must lose stability by divergence, while nonconservative ones may do so by flutter.

[2]For a clamped–clamped pipe, the dynamics is qualitatively similar, but the critical values of u are higher [see Païdoussis (1998)].

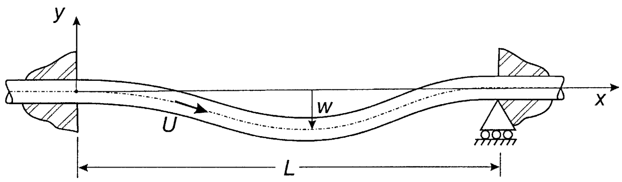

Fig. 1. Diagram of a clamped–clamped pipe, with a freely axially sliding downstream support, conveying fluid.

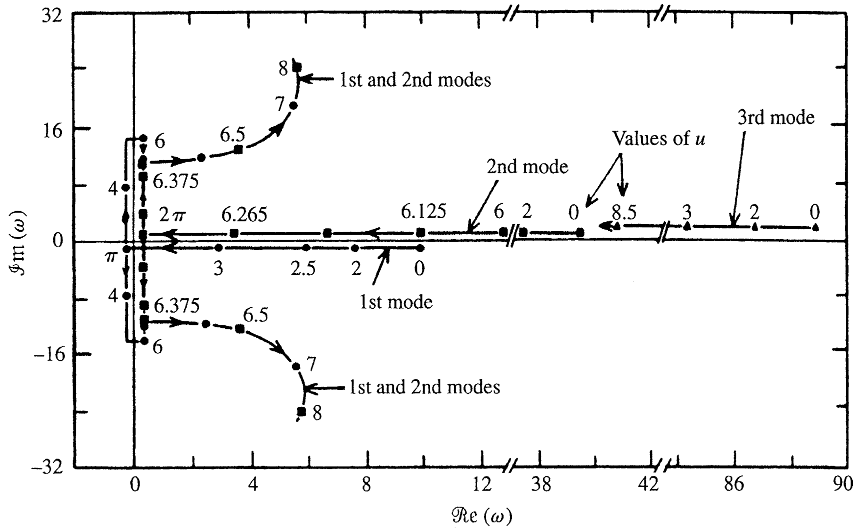

Fig. 2. Typical Argand diagram showing the variation of the dimensionless complex frequency ω as a function of the dimensionless flow velocity u, for a pinned–pinned pipe conveying fluid for $\beta = M/(M+m) = 0.2$, after Païdoussis and Issid (1974), Païdoussis (1998).

An ingenious attempt to resolve the paradox was made by Done and Simpson (1977), by considering the downstream end to be free to slide axially (Fig. 1), though prevented from moving laterally. Without reproducing this work here, the crux of the matter is that the momentum flux of the fluid issuing from the downstream end, MU^2, causes a mean contraction $\bar{c}$ of the pipe; the system then oscillates with contraction amplitude $\tilde{c}$, and hence with a lateral oscillation all along the pipe as well, without any net work being required to maintain the oscillation. In this, it should be remembered that, in the linear equation of motion, axial displacement does not appear explicitly — any more than it does in the Euler-Bernoulli equation for vibration of a beam with supported ends;[3] but its existence is nevertheless implied. Indeed, therein lies the main interest of this work in the present context: that it is possible for physically existent but mathematically absent (explicitly in the equation of motion) mechanisms to influence the dynamics of the system.

However, the definitive work on this was done by Holmes (1977, 1978) who undertook a nonlinear analysis of the problem. Indeed, this is *a must*, since linear theory is strictly applicable only up to the first loss of stability. First, a finite-dimensional analysis was undertaken (Holmes, 1977). As predicted by linear theory also, there is no restabilization after divergence if dissipation is taken onto account. The secondary bifurcation corresponding to the coupled-mode linear flutter threshold gives rise to an unstable solution, which therefore does not materialize, as shown in Fig. 3(c). The only stable attractors are the two sinks generated at $u = \pi$, shown in Fig. 3(b) and (c). Although there could be local oscillation about the origin, as shown diagrammatically in Fig. 3(d), eventually the system is captured by one of the sinks. For a system with zero dissipation (a Hamiltonian system), this is confirmed by computations, as shown in Fig. 4. However, the oscillation observed in Fig. 4 is pathologically nonrobust: the slightest amount of damping kills it; hence,

[3]Of course, axial extension *does* appear explicitly in nonlinear analyses of the system; e.g., if the two ends are positively fixed (i.e., axial sliding is prevented also), transverse deformation gives rise to axial extension and hence to deformation-induced tension, accounted for in the equations.

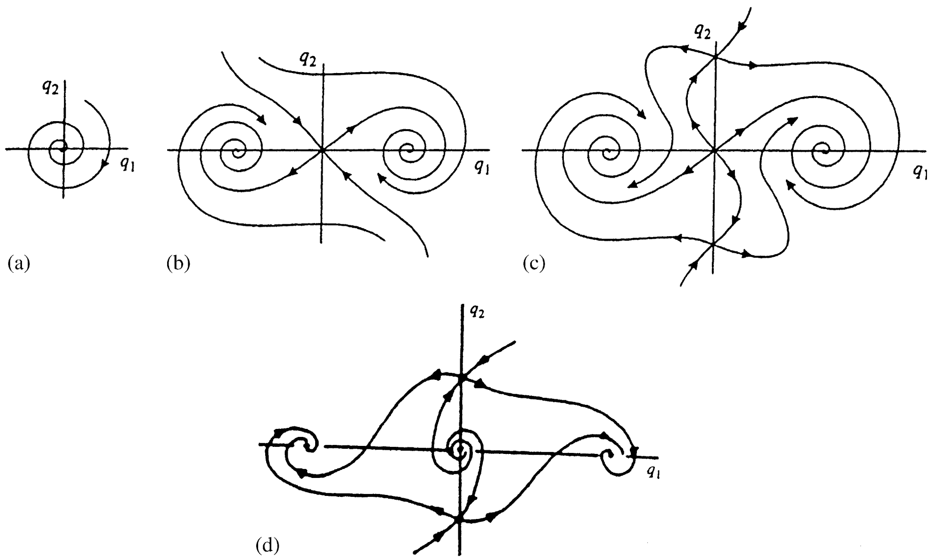

Fig. 3. A qualitative picture of the bifurcations of a pinned–pinned pipe conveying fluid, showing the vector field projected on the plane of the two generalized coordinates, $\{q_1, q_2\}$: (a) for the stable system, $u<\pi$; (b) after the first divergence at $u=\pi$; (c) after the second divergence at $u=2\pi$ [after Holmes (1977)]. (d) Schematic showing the possible existence of transient flutter for $u>2\pi$ [after Holmes (1978)]; (Païdoussis, 1998).

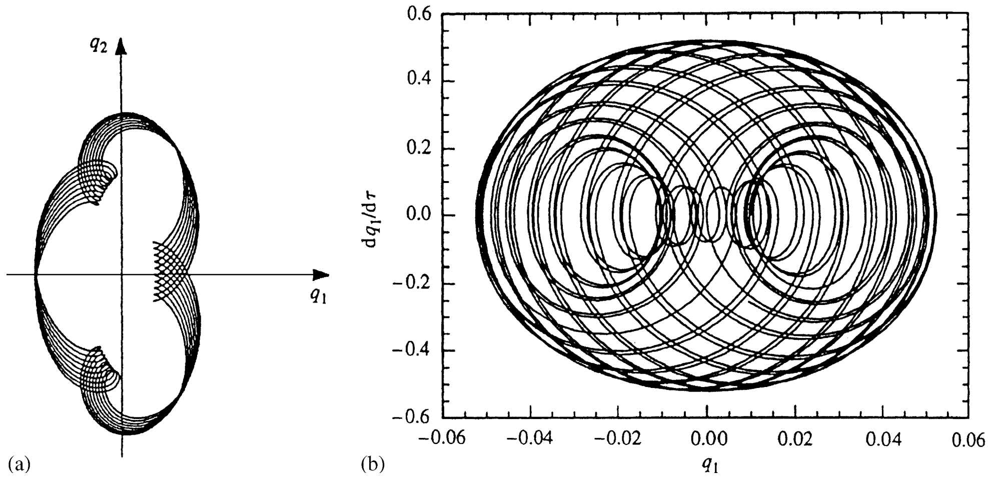

Fig. 4. (a) A 'limit cycle' in the $\{q_1, q_2\}$ plane for a pinned–pinned pipe conveying fluid with $u = 2.025\pi$ and zero dissipation; (b) phase-plane plot of the Hamiltonian system (Païdoussis, 1998).

coupled-mode flutter can only exist as a transient damped oscillation. Indeed, post-divergence flutter has never been observed experimentally.

Next, Holmes (1978) undertook an infinite dimensional analysis, assessing stability via the Lyapunov second (direct) method, proving unequivocally that post-divergence flutter is impossible. The title of the paper, "Pipes supported at both ends cannot flutter", started a remarkable trend: to state the main conclusion of the paper in its title.[4]

Details on all of the foregoing may be found in Païdoussis (1998).

[4]Particularly useful for overworked executives, who could thus digest the gist of a paper between the *de rigueur* champagne and the *hors d'oeuvres* on a first-class flight!

2.2. Post-divergence flutter of shells in contact with axial flow

The question is whether the foregoing also applies to shells conveying fluid.[5] Linear theory predicts divergence, followed by coupled-mode flutter, as shown for instance in Fig. 5, at $\overline{U} = 0.580$ and $\overline{U} = 0.607$, respectively; [6] for water-flow (Weaver and Unny, 1973) the two critical flow velocities are more widely separated.

In experiments with air-flow, the system was found to lose stability by flutter directly (Païdoussis and Denise, 1972); i.e., the system appeared to lose stability by flutter. This was explained at the time by theorizing that the onset of divergence precipitated (induced) flutter, and hence only the latter was observed.

Later, after experiments with annular flow were conducted (El Chebair et al., 1989), an alternative explanation became more plausible: that the observed flutter was a manifestation of *dynamic divergence*. Since in Païdoussis and Denise's experiments the elastomer (rubber-like) shells used were very pliable, the amplitudes of motion in the observed flutter were quite large. It was therefore reasoned that, if the system experienced divergence in, say, the $n = 2$ mode [see Fig. 6(a)], the collapse of the cross-section from circular to quite flat quasi-elliptical would severely obstruct the flow. The resultant build-up of pressure would force the shell open and throw it into the azimuthally opposite form in the same mode, as shown in Fig. 6(b). A succession of these deformations at a fairly regular frequency would then be indistinguishable from flutter. However, this was only a plausible scenario, and had to be tested both by theory and by experiment.

It was a considerable time later that a nonlinear model became available (Amabili et al., 1999a–c). This model is based on the Donnell nonlinear shallow shell equations (valid for very thin shells and $n \geqslant 4$ or 5) and the linear fluid-structure interaction model of Païdoussis and Denise. Typical results for a shell with simply supported ends are shown in Fig. 7. The system loses stability by a strongly subcritical pitchfork bifurcation at dimensionless flow velocity $V \simeq 3.3$; all subsequent bifurcations lead to static solutions, both stable and unstable. This lends weight to the conjecture that the observed flutter must in fact be an oscillatory (dynamic) divergence. [The results in Fig. 7 are without companion mode participation; with it, the dynamics is more complex and interesting, but the foregoing general conclusion is not altered (Païdoussis, 2003a).]

Further support to the hypothesis that the observed flutter is indeed a dynamic divergence was provided by more recent experiments by Karagiozis et al. (2003, 2004, 2005), using stiffer shells made of aluminum or plastic (PET) conveying water. In this case, the shells lost stability by static divergence, and no oscillatory motion was observed. Typical results are shown in Fig. 8. Here, $U_{cd} = 16.1\,\mathrm{m/s}$ with $n = 6$. The bifurcation is strongly subcritical; on decreasing the flow, stability was regained at $U_r = 9.1\,\mathrm{m/s}$. In this case, the maximum deformation was quite small compared to the radius (Fig. 8) and hence there was no substantial constriction to the flow as a result of the divergence (buckling).

With these recent observations, a reconciliation becomes possible between theory and experiment, insofar as loss of stability is concerned: (i) for sufficiently stiff shells, loss of stability is indeed by divergence, as predicted by theory and as observed in the experiments; (ii) for sufficiently pliable shells, on the other hand, the experimental divergence is dynamic (oscillatory), as a result of the large deformation of the shell and attendant obstruction to the flow. This is further reinforced by the fact that, for external (annular) flow over very pliable shells, static divergence of large amplitude has been observed (El Chebair et al., 1989; Païdoussis, 2003a; Karagiozis et al., 2005), since in this case no obstruction to the flow is generated by the inward collapse of the shell.

However, does post-divergence flutter exist at higher flow velocities, as predicted by linear theory? The nonlinear theory of Amabili et al., (1999a–c) suggests that it does not. However, in that theory a purely potential flow model is utilized. Yet, it has been found that viscous forces can be responsible for post-divergence flutter, as discussed in Section 2.3. Furthermore, it has been shown both for cylinders in axial flow (Païdoussis et al., 1990) and for models of pulmonary fluid–structure interaction that, instead of the system losing stability by divergence, it can do so by flutter directly (Grotberg and Reiss, 1982, 1984; Païdoussis, 2003a); it is realized, of course, that this is not the same as post-divergence flutter, but it is mentioned here to indicate that the effect of viscous flow and the corresponding stress resultants in the solid can have a significant effect on stability.

The matter could have been decided experimentally, if our experimental systems, both for external (annular) air flow and for internal water flow, could allow us to reach flow velocities, say, three times those giving rise to divergence. However, this is not the case, and so the definitive validation of the nonexistence of post-divergence flutter of shells subjected to internal or external axial flow remains elusive.

[5] Although the discussion here is confined to shells conveying fluid, it is equally applicable to shells immersed in external axial flow.

[6] The dimensionless flow velocity in this case is $\overline{U} = \{E/[\rho_s(1 - \nu^2)]\}^{1/2}$, where ρ_s is the density of the shell material and ν is the Poisson ratio.

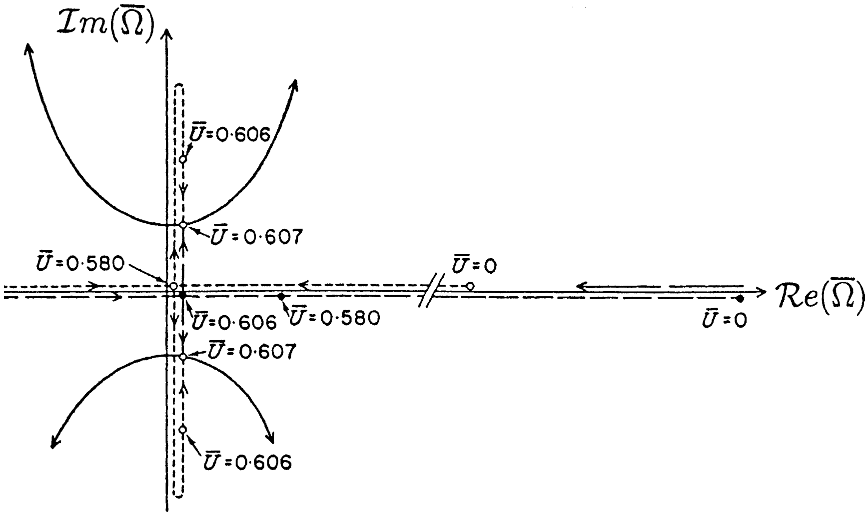

Fig. 5. Argand diagram of the dimensionless frequency $\overline{\Omega}$ as a function of the dimensionless flow velocity $\overline{U}$ for a clamped–clamped shell conveying fluid for $n = 2$ and $m = 1, 2, 3$ (Païdoussis and Denise, 1972; Païdoussis, 2003a).

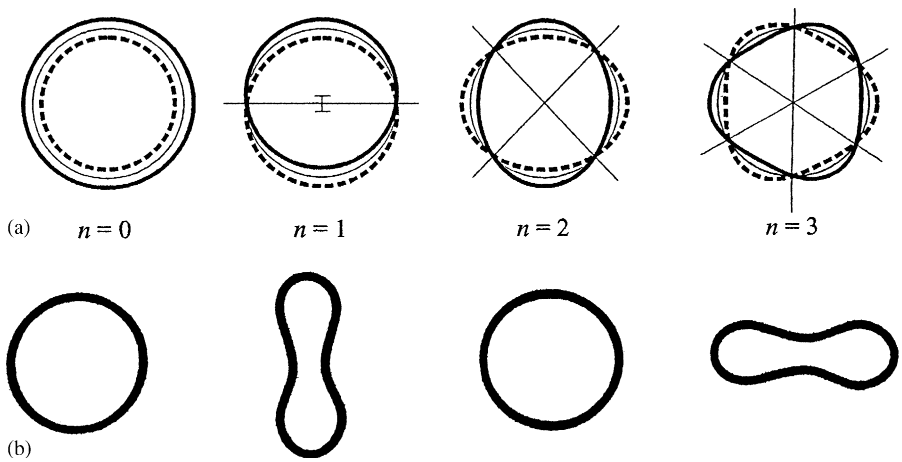

Fig. 6. (a) Circumferential mode shapes associated with $n = 0$, 1, 2 and 3. (b) The scenario for dynamic divergence, showing the collapse of the circular cross-section into an exaggerated $n = 2$ shape constricting the flow, followed by re-inflation and collapse in the azimuthally 180° out-of-phase configuration.

2.3. Cylinders in axial flow

The simplest form of the linear equation of motion of a cylinder in axial flow is

$$EI\,\frac{\partial^4 w}{\partial x^4} + M\,U^2\frac{\partial^2 w}{\partial x^2} + 2M\,U\frac{\partial^2 w}{\partial x \partial t} + F_v + (M+m)\frac{\partial^2 w}{\partial t^2} = 0, \tag{4}$$

where EI is the flexural rigidity and m the mass of the cylinder per unit length, U the flow velocity, and M the fluid added mass per unit length; F_v represents the viscous terms associated with longitudinal and transverse viscous forces along the beam,

$$\begin{aligned} F_v = &-\left\{\tfrac{1}{2}\rho\,D\,U^2\,C_T\left[\left(1-\tfrac{1}{2}\delta\right)L - x\right]\right\}\frac{\partial^2 w}{\partial x^2} \\ &+\tfrac{1}{2}\rho\,D\,U\,C_N\left(\frac{\partial w}{\partial t} + U\,\frac{\partial w}{\partial x}\right) + \tfrac{1}{2}\rho\,D\,C_D\,\frac{\partial w}{\partial t}; \end{aligned} \tag{5}$$

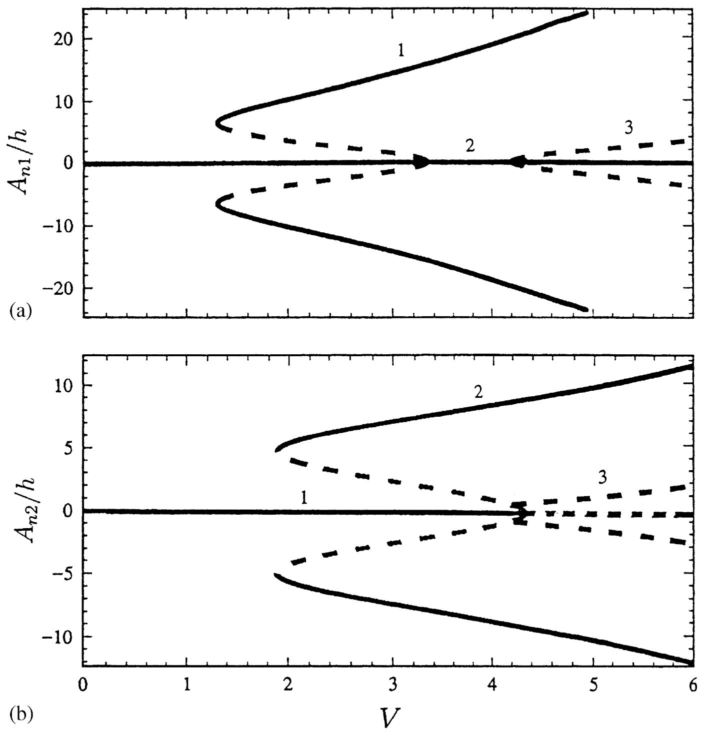

Fig. 7. (a) Nonoscillatory dimensionless modal amplitudes A_{n1}/h (for $n = 5, m = 1$), where h is the shell thickness versus the dimensionless flow velocity $V = U/[(\pi^2/L)(D/\rho_s h)^{1/2}]$, $D = Eh^3/[12(1 - \nu^2)]$, with a small amount of dissipation and without companion mode participation; (b) the same for $A_{n2}/h\,(n = 5, m = 2)$; after Amabili et al., (1999a), Païdoussis (2003a).

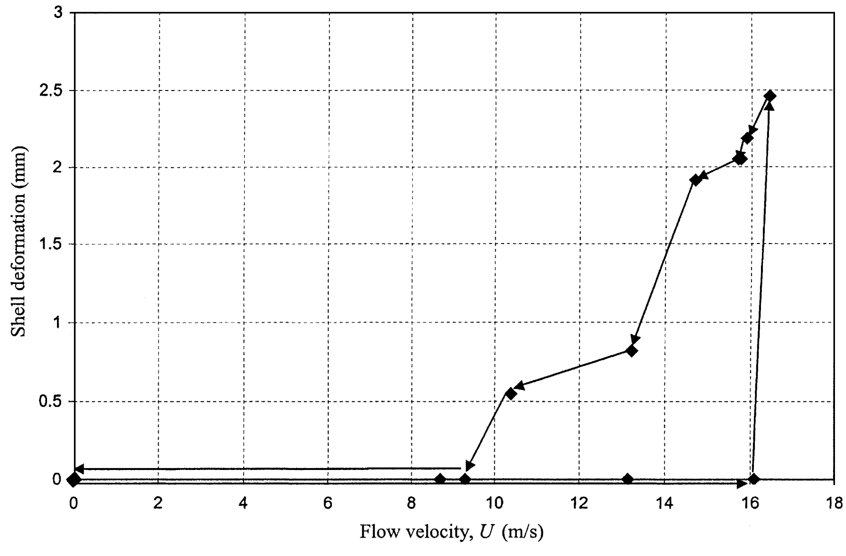

Fig. 8. Experimental bifurcation diagram for an aluminum shell (radius $a = 41.1$ mm, $L/a = 2.98$, $a/h = 300$) conveying water and surrounded by quiescent water, showing the static deformation of the shell and the development of strongly subcritical divergence (strong hysteresis), after Karagiozis et al. (2005).

C_T and C_N are the longitudinal and transverse viscous force coefficients, C_D a zero-flow viscous damping coefficient, D the cylinder diameter, and ρ the fluid density; $\delta = 0$ if the cylinder is free to slide axially at the downstream end, and $\delta = 1$ if it is positively fixed. Thus, Eq. (5) is identical to the equation for internal flow, Eq. (1), but for F_v. As mentioned before, in the internal flow case, viscous effects cancel out; in the external flow case, they do not — and hence F_v arises (Païdoussis, 2003a).

In terms of dynamical behaviour, however, in this case both linear theory (Païdoussis, 1966a, 1973, 2003a) and nonlinear theory (Païdoussis, 2003a; Modarres-Sadeghi et al., 2004, 2005) predict the existence of post-divergence flutter of the cylinder in axial flow, and this was also observed in experiments (Païdoussis, 1966b, 2003a). This contrasts to the conclusion reached in Section 2.1 that, for pipes conveying fluid, post-divergence flutter is not possible.

Recalling that we are considering systems with supported ends, the work done by all the forces on the system in a period of oscillation is found to be

$$\Delta W = -\tfrac{1}{2}c_N(M\,U/D)\int_0^T\int_0^L \left(\dot{w}^2 + U\,\dot{w}w'\right)\mathrm{d}x\,\mathrm{d}t - \tfrac{1}{2}(M/D)\int_0^T\int_0^L \left(c^*\,\dot{w}^2 - c_T\,U^2\,\dot{w}w'\right)\mathrm{d}x\,\mathrm{d}t, \tag{6}$$

in which $c_N = (4/\pi)C_N$, $c_T = (4/\pi)C_T$, $c^* = (4/\pi)C_D$, and $(\ )' = \partial(\ )/\partial x, (\dot{\ }) = \partial(\ )/\partial t$. Hence, for instability,

$$-(c_N - c_T)U^2\int_0^L \overline{\dot{w}w'}\,\mathrm{d}x - (c_N\,U + c^*)\int_0^L \overline{\dot{w}}^2\,\mathrm{d}x > 0. \tag{7}$$

For internal flow, c_N, c_T and c^* are totally absent from the equation of motion, and hence $\Delta W = 0$, which is the case discussed in Section 2.1, concluding that coupled-mode flutter for internal flow is not possible.

For external flow, however, generally $\Delta W \neq 0$ and hence coupled-mode flutter is in principle possible, as per Eq. (7), and in fact flutter does occur. The fact that the integral of $\overline{\dot{w}w'}$ should be negative to give $\Delta W > 0$ implies the existence of a travelling wave component in the motion of the cylinder, as discussed in Païdoussis (2003a). Also, as first noted by Holmes (1977), $\Delta W > 0$ is possible thanks to the presence of terms involving $U^2(\partial w/\partial x)$ in the viscous terms in the equation of motion, which are absent in the case of the fluid-conveying pipe.

According to nonlinear theory, post-divergence flutter is not a coupled-mode flutter arising from the trivial equilibrium state, but a Hopf bifurcation emanating from the post-divergence branch of the solution (Modarres-Sadeghi et al., 2004, 2005; Païdoussis, 2003a). The "coupled-mode flutter" bifurcation point on the trivial equilibrium locus does of course also exist in nonlinear theory, but it gives rise to an unstable, nonphysical solution branch.

In the experiments, it has been observed that divergence is neither as violent nor of as large in amplitude as in the internal flow case (Païdoussis, 2003a). This no doubt is associated with the flow-induced tension and the lateral viscous forces acting on the cylinder, which increase with flow. Hence, this too accords with theory that the existence of post-divergence flutter is, one way or another, related to the viscous forces acting on the structure — absent in the internal flow problem.

2.4. Concluding comments

It goes without saying that the dynamical behaviour of a system beyond the threshold of the first instability encountered — and hence the question of existence of post-divergence flutter in the problem at hand — must be assessed via nonlinear theory. Although in some cases (e.g., as shown in Section 2.3) linear theoretical predictions are validated by nonlinear theory, it is just as likely that they be invalidated thereby (as discussed in Section 2.1).

Another way of testing the predictions of linear theory is, of course, by experiment (as, for example, in the case of a cylindrical beam with internal or external flow in Sections 2.1 and 2.3). However, what is particularly interesting in this regard in the work presented is the occurrence of loss of stability by dynamic divergence (for a shell with internal flow, in Section 2.2), which can easily be, and was initially, mistaken for flutter.

3. Stability of shells differently supported at their two extremities

3.1. A succession of conflicting findings on conservativeness of the system

In the 1980s a series of papers by Horáček and Zolotarev treated the dynamics of fluid-conveying shells with different, i.e. mismatched or "asymmetric", end-support conditions; e.g., shells simply supported at the upstream end

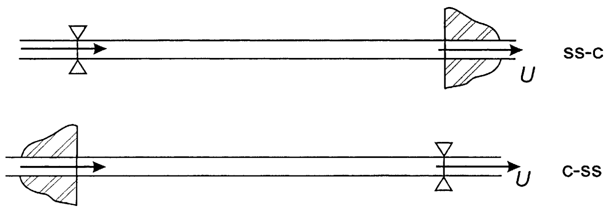

Fig. 9. Definition of ss-c and c-ss shells conveying fluid.

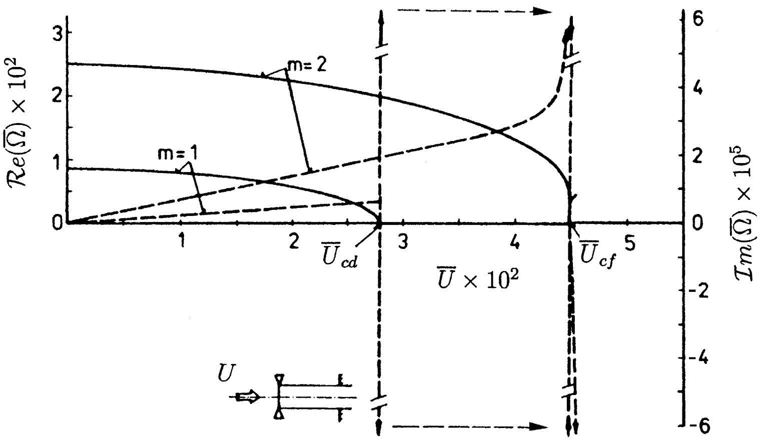

Fig. 10. The real (—) and imaginary (– – –) parts of the dimensionless eigenfrequencies $\overline{\Omega}$ of a ss-c shell conveying fluid for $n = 6, m = 1$, as a function of the dimensionless flow velocity $\overline{U} = U(\rho_s/E)^{1/2}$; after Horáček and Zolotarev (1984), Païdoussis (2003a).

and clamped at the downstream one (ss-c, for short); see Fig. 9. It was found [see, e.g., Horáček and Zolotarev (1984)] that ss-c shells conveying *inviscid* fluid exhibit flow-induced damping for all $U > 0$ (see Fig. 10), before eventually losing stability by divergence. Even more radical was the prediction that shells clamped at the upstream end and simply supported at the other end (c-ss, for short) exhibit *negative flow-induced damping*; i.e., they are unstable, again for all $U > 0$!

On the other hand, calculations by Païdoussis et al. (1993) showed that both ss-c and c-ss systems behave as conservative systems, i.e., there is *zero* positive or negative flow-induced damping up to the onset of divergence. Indeed, the work done in a presumed period of oscillation is

$$\Delta W = -\rho\pi a^2 \, \mathscr{I}(n,\lambda) \, U \int_0^T \left[\left(\frac{\partial \overline{w}}{\partial t}\right)^2 + U \left(\frac{\partial \overline{w}}{\partial x}\right)\left(\frac{\partial \overline{w}}{\partial t}\right) \right]_0^L \mathrm{d}t, \tag{8}$$

where ρ is the fluid density, U the flow velocity, a and L the shell radius and length, respectively, $\mathscr{I}(n,\lambda)$ a functional of Bessel functions dependent on the circumferential and axial wavenumbers, n and m respectively, and $\overline{w}$ the radial shell displacement corresponding to $w(x, \theta, t) = \overline{w}(x, t)\cos n\theta$. Clearly, in the case of supported ends, whether c-ss or ss-c, one must have

$$\Delta W = 0. \tag{9}$$

So, again, the quandary arises as to how can this be so, yet at the same time the system be subject to flow-induced damping, positive or negative?

At first, it was thought that Vol'mir's semi-membrane shell theory initially used by Horáček and Zolotarev was the culprit, but they later found the same kind of results with the fuller Goldenveizer–Novozhilov shell theory. It was then thought that the problem must be numerical. [It must be stressed here that the predicted damping is rather small; the damping ratio is $\zeta = \mathscr{I}m(\overline{\Omega})/\mathscr{R}e(\overline{\Omega})$, so that $\zeta \sim \mathcal{O}(10^{-3})$ in Fig. 10. To this end, an extensive study was undertaken by Misra et al. (2001).

However, before that, it occurred to the author that, if this happens with shells, then why not with c-ss and ss-c *pipes* (governed by the Euler–Bernoulli beam equation)? The initial set of calculations produced results similar to Horáček and Zolotarev's for shells! Thus, the c-ss system behaves as a nonconservative one, with $\mathscr{I}m(\omega)<0$ for all $U<U_{cd}$, suggesting that flutter should occur from arbitrarily small U up to the onset of divergence. Yet, these c-ss and ss-c systems are conservative by conventional wisdom, as much as ss-ss and c-c systems are! *This* paradox was resolved via more careful calculations. The comparison functions used in the Galerkin-type solution were the beam eigenfunctions, the eigenvalues of which had been computed to "only" 6 significant figures. Calculations with 9 significant-figure accuracy showed the c-ss to be totally conservative and stable up to $U = U_{cd}$. (Interestingly, the ss-c system was found to be conservative even with 4 significant-figure accuracy, indicating that the numerics in this case are much less sensitive to imprecision.) So, elementary stability theory was *not* turned on its head, and the virtuous could continue sleeping peacefully at night! This also gave added credence to the supposition that the cause of the Horáček and Zolotarev "rogue" findings was numerical inaccuracy.

Typical results from the Misra et al. (2001) study for a c-ss shell conveying fluid are shown in Fig. 11. The results in Fig. 11(a) suggest the same behaviour as found for the pipe; i.e., as the precision is increased, the values of $\mathscr{I}m(\overline{\Omega})$ are diminished, implying they would vanish if precision were increased sufficiently; moreover, for the ss-c system with U replaced by $-U$ one obtains $\mathscr{I}m(\overline{\Omega})\sim\mathscr{O}(10^{-8})$ from Fig. 11(b), effectively zero. Similar results were obtained with calculations using a Fourier-transform type of solution. Thus, between these findings and Eqns. (8) and (9), Misra et al. (2001) concluded that the problem had been resolved: the c-ss and ss-c systems are indeed conservative.

The same conclusion was reached by Amabili and Garziera (2002) in a similar way as in the work leading to Eqs. (8) and (9), i.e., via energy considerations. The energy associated with the perturbation flow field was determined and split into three components: a reference kinetic energy, a maximum potential energy and a maximum gyroscopic (Coriolis-acceleration-related) energy. It is this latter that could potentially lead to $\mathscr{I}m(\overline{\Omega})\neq 0$ before divergence, and hence to nonconservativeness of the system. However, Amabili and Garziera show that the system is "conservative for any combination of boundary conditions with restrained displacement at the shell ends", i.e., for $\overline{w}(x,t)$ zero at both ends. Moreover, numerical results are also provided to support this conclusion. The flow-induced damping legible from the graphs in Amabili and Garziera (2002) looks to be zero, as did that in the Païdoussis et al. (1993) results. The question is: zero to what accuracy? The answer (Amabili, 2004) is quite stunning: $\zeta<10^{-10}$!

Hence, one would have thought, between the Misra et al. (2001) and the Amabili and Garziera (2002) results, the conclusion is clear: c-ss and ss-c systems are indeed conservative.

However, the paradox was resurrected by newer calculations by Horáček and Zolotarev (2002): with very small L/a, the values of $\mathscr{I}m(\overline{\Omega})$ are significantly larger, too large to be explained as due to numerical imprecision. Thus, in one case, for $L/a = 2$, $n = 5$, $m = 1$, the flow-induced damping is found to be $\zeta \simeq \pm 6 \times 10^{-4}$ at $U = 10\,\text{m/s}$; assuming a linear variation with U (cf. Fig. 10), at $U_{cd} = 332\,\text{m/s}$ this would give $\zeta \simeq \pm 2 \times 10^{-2}$, or approximately ten times larger than in Fig. 10. Furthermore, more recently, Zolotarev (2004) provided a proof that a fluid-conveying shell with mismatched end supports is generally a nonconservative system. Hence, it would appear that we are back to square one; not quite, though.

In view of the foregoing work by others, the main question now is not so much whether the system is conservative or not. Rather, the key question is: why are the results obtained by Horáček and Zolotarev (2002) and Zolotarev (2004) different from those obtained by the others, and hence their conclusions regarding conservativeness also different?

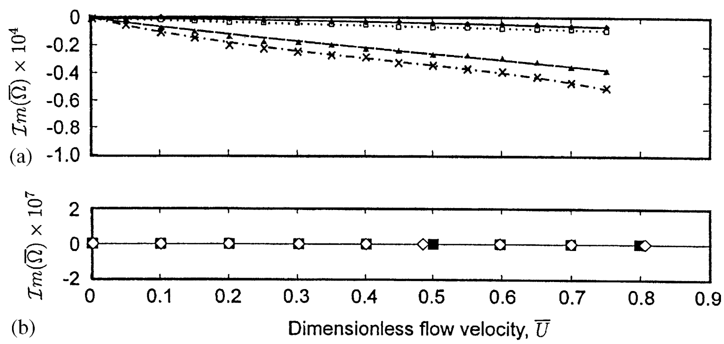

Fig. 11. (a) The imaginary component of the eigenfrequencies for $n = m = 2$ of a c-ss shell conveying fluid, for different accuracies in the axial wavenumber and program tolerances in the travelling-wave analysis: ×, double precision; △, long double precision; ■, 10^{-6} tolerance; ⋄, 10^{-8} tolerance. (b) The same as in (a), but with the problem recast as a ss-c shell and the flow direction reversed; after Misra et al. (2001), Païdoussis (2003a).

3.2. Some thoughts towards resolution of the issue

A clue towards the resolution of the problem was provided by some earlier work by Zolotarev (1987a, 1987b) [see also Horáček and Zolotarev (2002)] for a fluid-conveying shell supported by springs at its downstream end, as shown in Fig. 12(a). By setting the spring constants equal to zero or to infinity, all standard boundary conditions may be obtained. Thus, a c-ss shell is obtained if $k_4 = 0$, $k_2 = k_3 = \infty$; if additionally the axial spring constant $k_1 = \infty$, one obtains one of "the classical" conditions for a simply supported end: no axial sliding permitted.

In Fig. 12(b) are shown the results for $k_2 = k_3 = \infty, k_4 = 0$, and k_1 varied. It is seen that for $k_1 = \infty$, $\delta \equiv 2\pi \mathscr{I}m(f)/\mathscr{R}e(f) = 0$; hence $\mathscr{I}m(f) = 0$, and the system is conservative. For $k_1 \neq \infty$, however, the system is nonconservative, and flutter is predicted, starting at infinitesimal flow velocity. Thus, allowing axial extension at the downstream end evidently permits energy to flow from the fluid to the shell, thereby inducing flutter. This is clearly reminiscent of the Done and Simpson (1977) findings referred to in Section 2.1. The reader is reminded here of the conjecture made concerning the influence of mathematically explicitly absent (but implied) physical mechanisms influencing the dynamics.

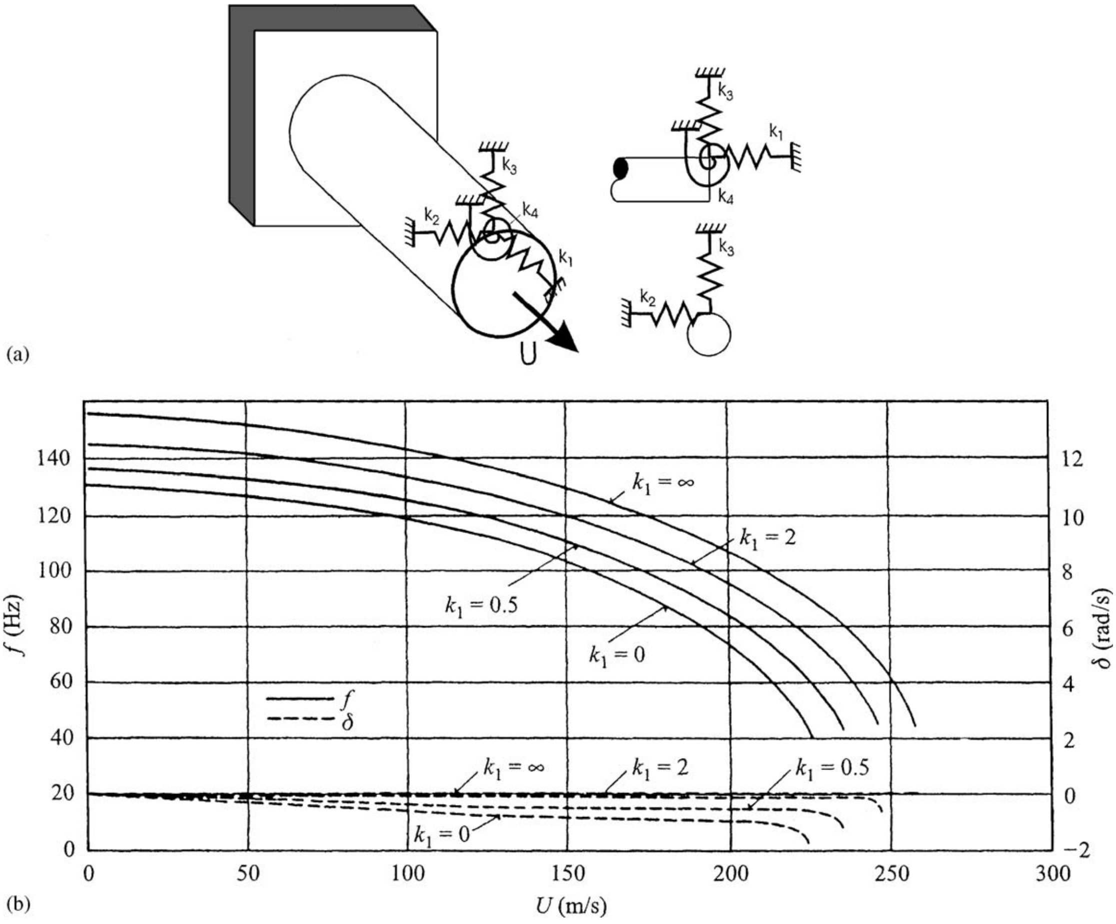

Fig. 12. (a) A shell clamped upstream and spring-supported downstream, conveying fluid. k_1 is in the axial direction, k_2 in the circumferential direction, and k_3 is in the radial one; k_4 is a torsional spring associated with $\partial w/\partial x$. These are further clarified in the side-view and cross-sectional view on the right. The four differernt sets of springs are distributed along the downstream circumference of the shell, though here, for clarity, only one spring of each kind is shown, at one circumferential location. (b) The effect of the axial spring constant k_1, on the frequency (—) and logarithmic decrement (– – –) of the $n = 6$, $m = 1$ eigenfrequency as a function of flow velocity U; after Zolotarev (1987a,b), Païdoussis (2003a).

It may thus be proposed that the work done in a period of oscillation is

$$\Delta W = -\mathscr{M}_1 U \int_0^T \left(\dot{\overline{w}}^2 + U\dot{\overline{w}}\,\overline{w}'\right)\Big|_0^L \mathrm{d}t + \mathscr{M}_2 U \int_0^T \left(\dot{\overline{u}}^2 + U\dot{\overline{u}}\,\overline{u}'\right)\Big|_0^L \mathrm{d}t, \tag{10}$$

where the first term is the right-hand side of Eq. (10), written somewhat differently, and is related to radial shell movements, while the second term is related to axial movements, even though such a term does not arise from the equations of motion in the same way as the first one does. Such a term does exist, however, in a *nonlinear analysis* of a pipe conveying fluid [see Eqs. (5.36a,b) in Païdoussis (1998)]. For a shell with supported ends, the first term is zero, as already discussed. If one of the ends is sliding, however, the second term is nonzero, and ΔW is positive or negative for small U accordingly as the downstream end (c-ss shell) or the upstream end (ss-c shell) can slide.

In Zolotarev's (1987a,b) work, reproduced also in Horáček and Zolotarev (2002), the Goldenveizer–Novozhilov shell theory is used, which is similar to Flügge's, utilized by Païdoussis et al. (1993) and Misra et al. (2001), in both of which $u = 0$ was imposed at both ends;[7] this corresponds to $k_1 = \infty$ in Fig. 12(a,b). The fluid-dynamic model is also similar. Therefore, to this extent, the results are consistent: if $u(0) = u(L) = 0$, the system is conservative. If either $u(0)$ or $u(L) \neq 0$, then it is nonconservative: for $u(0) \neq 0$, the system is stabilized by the flow; for $u(L) \neq 0$, the system is destabilized by the flow, and in the absence of dissipative forces becomes unstable at infinitesimal U. If both $u(0)$ and $u(L)$ are nonzero but equal via identical axial springs at the two ends; i.e., if the end-supports are *matched*, then by Eq. (10) it is seen that the effect cancels out, and the system is again conservative.

Next, we comment on the aforementioned proof of nonconservativeness provided by Zolotarev (2004). For simplicity and tractability in the proof, Vol'mir's semi-membrane theory for the shell was used, which is similar to Donnell's shallow shell theory. Thus, there is a single equation of motion, involving $w(x, \theta, t)$, the radial deflection. However, once $w(x, \theta, t)$ is obtained, one can compute the axial deflection $u(x, \theta, t)$ and the circumferential deflection $v(x, \theta, t)$. In the linear Donnell theory, $u(x, \theta, t)$ may be determined via

$$\nabla^2 u + \nu w''' - w'^{\cdot\cdot} = 0, \tag{11}$$

where $(\ )' = a\partial(\ )/\partial x$, $(\ )^{\cdot} = \partial(\ )/\partial\theta$, a being the shell radius. To solve the equation of motion for c-ss or ss-c shells, boundary conditions on w are imposed, but as seen in Eq. (11) this does not necessarily mean that $u(0) = u(L) = 0$. Hence, Zolotarev's proof does not negate the foregoing, which in effect states that the system is nonconservative only if $u(0) \neq 0$ or $u(L) \neq 0$.

All of the above cannot be said to constitute a proof and a definitive explanation. Past experience with this problem counsels caution. Indeed, the arguments associated with Eq. (10) may appear to be rather fanciful. The following provides an alternative explanation for the nonconservative behaviour exhibited in the Horáček and Zolotarev results.

3.3. *On the effect of the fluid boundary conditions and methods of solution*

In the foregoing, the discussion focused on axial sliding of the shell at the supports and attendant inlet/outlet axial fluid acceleration effects. More generally, however, there is a difference in the fluid boundary conditions upstream and downstream between the various analyses. In Païdoussis et al. (1993) and Misra et al. (2001), either the Fourier transform generalized-force method is used whereby the flow perturbations are zero beyond the shell domain [0, L], or the travelling wave form of the solution is adopted [as initially proposed by Païdoussis and Denise (1972)] where the waves continue up- and downstream indefinitely. Thus, in the travelling wave solution, the real domain is [0, $2L$], which for a simply-supported end encompasses the lowest waveform of wavelength $2L$ with a transient node at $x = L$; see Fig. 13. Then, by considering only antisymmetric modes with respect to $x = 0$ and $x = L$, as in Fig. 13, the analysis can be carried out over only half of that domain, i.e., for [0, L]. However, what happens beyond, for $x < 0$ and $x > L$, *is* of importance. How does the fluid behave before the entry and beyond the exit? Does one need to specify "outflow models" (Païdoussis, 2003a, Section 7.3.2)? Furthermore, how important is it for the fluid model to satisfy a true clamped-end condition, as opposed to reasonable approximations thereof?

In a related problem, that of a rocking rigid cylinder in annular flow, the specification (via Heaviside functions) or not of fluid boundary conditions was found to play an important rôle on the existence/nonexistence of flutter (de Langre et al., 1992; Païdoussis, 2003a, Section 11.5.2(f)). Not specifying the fluid dynamics beyond the domain [0, L] is not equivalent to the vanishing of the fluid-dynamic forces related thereto.

[7] More precisely, in Misra et al. (2001), this was one of the two sets of axial boundary conditions used; see also Wong (2000).

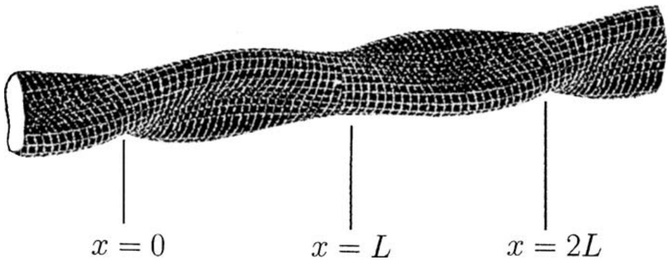

Fig. 13. The waveform (for $n = 2$) propagating in an infinite shell periodically supported at $0, \pm rL$, $r = 1, 2, \ldots, \infty$, showing that the fundamental domain is [0, 2L] rather than [0, L].

It must also be said that, although this author cannot fault the nitty-gritty of Zolotarev (2004) proof of nonconservativeness of the c-ss and ss-c systems, the proof is based on *a specific shell theory and method of solution*; i.e., it is based on (i) the Vol'mir/Donnell-type one-equation shell theory, and (ii) the Païdoussis and Denise fluid-dynamics model, in which solutions of the type $\{u, v, w\} = \sum_{j=1}^{\infty} \{A_j, B_j, C_j\} \exp\left[i(\lambda_j x/a + n\theta + \Omega t)\right]$ are used. The limitations of (i) have been discussed in Section 3.2. We now discuss item (ii). Because only eight shell boundary conditions are available to determine the infinite set of constants A_j, B_j, C_j, the summation is truncated at $j = 8$. To test convergence of the $j = 8$ solution, a solution with $j = 4$ was also obtained by Païdoussis and Denise, and the results with $j = 4$ were found to be close to those with $j = 8$, but by no means identical. In the Vol'mir/Donnell-type of theory, $j = 4$ truncation is *necessary* since only two boundary conditions are available for each end $\left(w = 0,\ \partial^2 w/\partial x^2 = 0\right)$. It is obvious, therefore, that the solution, whether truncated at $j = 4$ or 8, is fundamentally *approximate*, no matter how accurate the computation is made to be; the conclusions based thereon should therefore be taken with caution.

The same concern on the approximate nature of the solution has been expressed obliquely but more fundamentally by Amabili and Garziera (2002). In the travelling wave solution, one assumes that the fluid domain is axially infinite. The separation of variables technique used therefore implicitly requires that $\overline{w}(x, t)$ be defined for any $x \in (-\infty, \infty)$, which in turn necessitates that $\overline{w}(x, t)$ be a periodic function of x, with main period $2L$, as discussed before; it is through this artifice that velocity and pressure boundary conditions do not have to be imposed at the ends of the shell. In Amabili and Garziera (2002), Amabili et al., (1999a–c) and Païdoussis (2003a), for instance, the ''mathematical trick'' is used of taking $\overline{w}(x, t) \propto \sum_j \sin(j\pi x/L) \exp(i\Omega t)$, which is indeed x-periodic.[8] On theother hand, solutions involving $\sum_j (i\lambda_j x/a)$, as in Zolotarev's proof and calculations,[9] are not x-periodic generally, since not all the λ_j used are real, and no inlet/outlet fluid boundary conditions are used either; hence, such solutions are at best approximate.

The effect of an approximative/imperfect solution in this regard is tantamount to not satisfying fully the necessary fluid boundary conditions, which, as discussed three paragraphs earlier, is very important for the correct prediction of the dynamics. Furthermore, this effect will be more pronounced for shorter shells, which may explain the ''greater nonconservativeness'' of the results obtained by Horáček and Zolotarev (2002) for shorter shells.

Therefore, looking at the Zolotarev type of solution in either of these two ways, there is no doubt that it is fundamentally approximate, and hence it is impossible to decide through it whether the eigenfrequencies of the fluid-conveying shell are 100% real,[10] no matter how accurate the computations or precise the analysis based thereon, and hence whether the system is conservative or otherwise.

[8]The Amabili and Garziera solution approach is rather intricate and quite ingenious. A clamped end is represented by a simply supported one constrained by rotational springs, the stiffness of which are eventually made large enough to approximate a true clamped condition. Furthermore, the coefficients of the $\sin(j\pi x/L)$ terms in the solution are not determined by using the boundary conditions, but by minimization of the system energy via the Rayleigh–Ritz method. In this way, the summation over j can have as many terms as desired for improved accuracy. The functions used must of course satisfy the geometric boundary conditions. The zero displacement condition is the only geometric boundary condition in this scheme (automatically satisfied by the sine functions). In the spring-constrained system, we generally do not have the zero-slope geometric boundary condition, but a natural one of the type Moment $= k(\partial w/\partial x)$; thus, $\partial w/\partial x = 0$ can be satisfied by making k sufficiently large (Amabili, 2004).

[9]Also used in some previous solutions [e.g., Païdoussis and Denise (1972)] for other than simply supported shells, as well as in the results based on travelling wave solutions by Misra et al. (2001).

[10]Here the reader is reminded of the effect of approximations on the beam-pipe problem discussed in the fourth paragraph of Section 3.1, leading to a fictitious nonconservativeness!

3.4. Difficulties in experimental verification

Before closing this section, it should be stated that deciding the question of conservativeness or nonconservativeness of the c-ss and ss-c systems by experimental means would be difficult. If the shell is fairly long ($L/a = 5 - 10$, say) Horáček and Zolotarev predict a rather small flow-induced damping, easily overshadowed by dissipative damping due to fluid viscous effects and material damping in the shell. If the shell is short ($L/a = 2$, say), a larger flow-induced damping is predicted, but imperfections in the end-supports and local flow irregularities in the neighbourhood of the connections of the shell to up- and downstream piping would not be negligible, and could contaminate sensitive damping measurements.

Thus, settling the question experimentally is fraught with practical difficulties, though it is not necessarily impossible.[11]

3.5. Summary

A *dénouement* of this quandary appears to have been reached. The weight of reliable evidence (notably energy considerations) shows the c-ss and ss-c systems to be conservative. The evidence to the contrary is less than wholly convincing, because it is based on more approximate methods of solution.

4. Stability of aspirating cantilevers

4.1. State of the question prior to 2004

For a cantilevered pipe discharging fluid, Eq. (2) leads to

$$\Delta W = -MU \int_0^T \left[\left(\frac{\partial w}{\partial t}\right)_L^2 + U \left(\frac{\partial w}{\partial t}\right)_L \left(\frac{\partial w}{\partial x}\right)_L \right] \mathrm{d}t \neq 0. \tag{12}$$

If $\overline{(\partial w/\partial x)_L(\partial w/\partial t)_L} < 0$, where the overbar indicates the long-time average, $\Delta W > 0$ is obtained, indicating that the pipe can gain energy from the fluid, and indeed this is the mechanism of loss of stability by flutter at sufficiently high U — say at U_{cf}. This also agrees with experimental observation (Bourrières, 1939; Benjamin, 1961; Gregory and Païdoussis, 1966).

It is interesting to remark that, if the fluid velocity is reversed, so that the fluid is not discharged at the free end, but it is aspirated instead, i.e., if U is replaced by $-U$, then the *reverse* dynamical behaviour is predicted (Païdoussis and Luu, 1985): the system is initially *unstable* ($0 < U < U_{cf}$), and at U_{cf} it regains stability! If dissipation is taken into account, this perplexing finding is modified, but not radically. Apart from its inherent fundamental interest, this has repercussions on ocean mining, e.g., of manganese nodules, where essentially a vacuum-cleaner tube sucks water from the bottom of the sea, together with the minerals (Fig. 14); refer, e.g., to Chung (1996), Deepak et al. (2001) and Xia et al. (2004). If contact is lost with the sea bottom, is this system unstable? From the above, it would appear so.

Attempts to prove the existence of this flutter experimentally (if not at infinitesimal flow because of dissipation in the pipe material and due to friction with the surrounding ambient fluid, then, let us say, at quite low flow velocities) "failed" in the following sense. The cantilevered pipe remained inert, as the water flow was increased, until it collapsed as a shell at a point close to the support, where the transmural pressure was greatest as a result of pressure drop in the internal flow. This was the case for all reasonably flexible pipes tested, pipes which could readily be made to flutter with discharging rather than aspirated fluid.

Thanks to David J. Maull of Cambridge University, this paradox was linked to the quandary that perplexed Richard Feynman in the late 1930s: does a rotary lawn sprinkler rotate backwards if the water were sucked rather than discharged?

The matter was thought to have finally been resolved via a more careful assessment of the problem, based on the "obvious" realization that the flows exiting and entering the free end of the pipe are not at all similar — obvious, that is, after the fact! So, replacing U by $-U$ does not capture the total picture. Indeed, it was shown that for the aspirated flow

[11]The caution expressed here is well founded. The author once said, in a plenary talk, that to demonstrate the existence of follower-force-induced flutter of a cantilever (Beck's problem) would require "a rocket engine mounted on the free end of a beam column, or something similar!" (Païdoussis, 1986), implying that this would be all but impossible. Yet, shortly after, Sugiyama et al. (1990) did just that (Païdoussis, 1998, Section 3.2.2)!

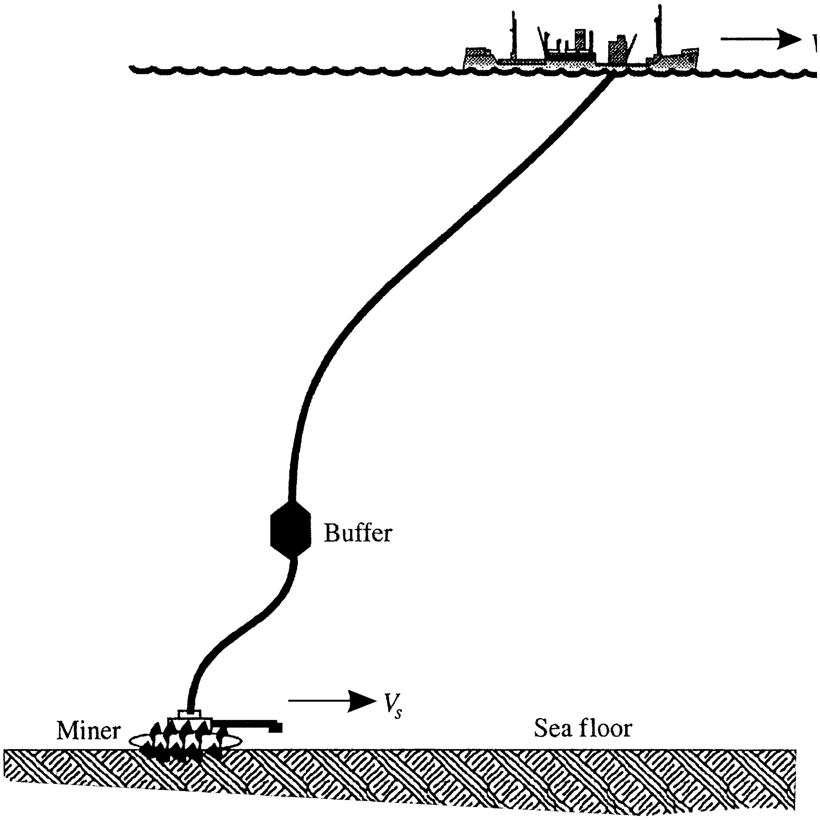

Fig. 14. The ocean mining system, involving an aspirating pipe; after Chung and Whitney (1983), Païdoussis (1998).

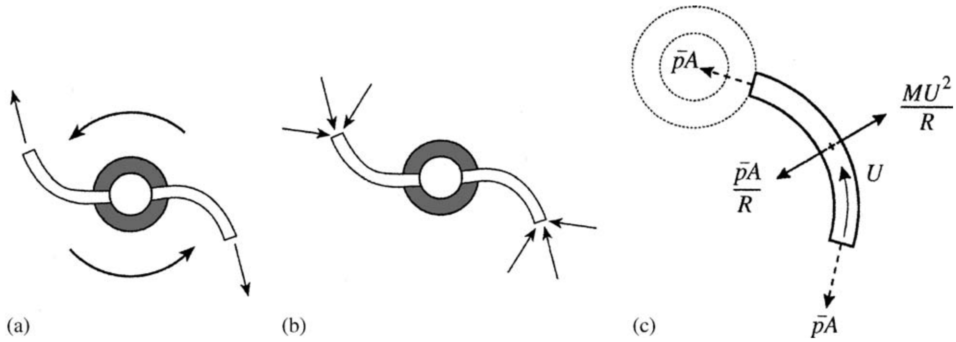

Fig. 15. (a) The discharging sprinkler, and (b) the aspirating sprinkler. (c) Negative pressurization and centrifugal forces on one arm of the aspirating sprinkler (Païdoussis, 1998).

there is a depressurization at the pipe inlet and throughout, equal to $\overline{p} = -MU^2/A$, A being the flow cross-sectional area. This effectively counteracts the MU^2 term due to the momentum flux, which is of cardinal importance in determining the dynamics — i.e., the third term in Eq. (13) in what follows. Hence, it was concluded — too fast as will be discussed in Section 4.2 — that no instability at all is possible for the aspirating cantilever. Similarly, for the same reasons (see Fig. 15), the sucking lawn sprinkler rotates neither backward nor forward!

This conclusion was supported by an experiment (Païdoussis, 1998, 1999) involving two nominally identical elastomer cantilevered pipes, hung vertical in a large tank, one discharging water and the other aspirating water. Both pipes were fitted at their free ends with identical light plastic 90° elbows. The two pipes were interconnected via a pump, so that the flow through them was the same. After the pump was switched on, the discharging pipe became bent as a result of the centrifugal force $M\,U^2/R$ at the 90° elbow, R being the radius of curvature. The aspirating pipe, however, after a brief transient, remained straight, since $M\,U^2/R$ is exactly cancelled out by the opposing $\overline{p}A/R$ term, with $\overline{p}A = -M\,U^2$.

Based on the results of this experiment and the discussion above it was concluded that "Aspirating pipes do not flutter at infinitesimally small flow", which *à la Holmes* was also the title of the paper (Païdoussis, 1999).

4.2. Lacunae and fresh doubts

Recently, fresh doubts have been expressed by Kuiper and Metrikine (2005) [see also Païdoussis (2004) and Païdoussis et al. (2005)] about the generality and correctness of some aspects of the Païdoussis (1999) work. Briefly, these may be summarized as follows:

(i) even if all centrifugal terms cancel out, the Coriolis term, which for a discharging pipe damps motions for small U, in the case of an aspirating pipe generates negative damping for flow velocities $-U$, thus potentially causing instability, essentially as originally asserted by Païdoussis and Luu (1985);
(ii) the result that $\overline{p} = -M\,U^2/A$, or $\overline{p} = -\rho\,U^2$, is doubtful since it contravenes Bernoulli's equation for the flow from a stagnant state far away to the pipe inlet, as will be detailed below; hence the centrifugal forces may not totally cancel out!

According to Bernoulli's equation, presuming a flow not quite like a pure sink flow at the intake, we can write $p_\infty + \frac{1}{2}\rho\,U_\infty^2 = p_o + \frac{1}{2}\rho\,U_o^2$, in which the subscript ∞ means "far away", and hence $U_\infty = 0$; the subscript o means "just facing the pipe" and, according to Kuiper and Metrikine (2005), also within the inlet. Hence, if as usual the pressure is measured relative to the ambient, we can write $p(L) \equiv p_L = p_o - p_\infty \equiv -\frac{1}{2}\rho\,U^2$, and $\overline{p}A = -\frac{1}{2}MU^2$, which represents a depressurization at the intake, but half as much as in Païdoussis (1999).[12] Note that this is the same as found, say, in Idel'chik (1986) for the pressure loss at a square-cut intake. Next, noting the aforementioned (in Section 2.1) balance between tension and pressure loss within the pipe, $\partial(T - pA)/\partial x = 0$, one realizes that only externally applied tension or pressurization play a role in the dynamics; hence, the aforementioned depressurization applies throughout the pipe, the equation of which, taking the flow velocity to be $-U$ within the pipe and taking into account the added mass of the surrounding fluid, becomes

$$EI\frac{\partial^4 w}{\partial x^4} - \left(\overline{T} - \overline{p}A\right)\frac{\partial^2 w}{\partial x^2} + M\,U^2\frac{\partial^2 w}{\partial x^2} - 2\,M\,U\frac{\partial^2 w}{\partial x \partial t} + (M + m + M_a)\frac{\partial^2 w}{\partial t^2} = 0, \tag{13}$$

where $\overline{T}$ and $\overline{p}$ are the mean tension and pressurization, and M_a the added mass due to the surrounding fluid per unit length. Thus, since $\overline{p} = p_L$ and $M = \rho A$, one obtains $\overline{T} - \overline{p}A = -\frac{1}{2}M\,U^2$, presuming $\overline{T} \simeq 0$, instead of $\overline{T} - \overline{p}A = -M\,U^2$ as in Païdoussis (1999), and the centrifugal forces do not totally cancel out. In fact, one obtains

$$\Delta W = M\,U\int_0^T \left[\left(\frac{\partial w}{\partial t}\right)^2 - \frac{1}{2}U\frac{\partial w}{\partial x}\frac{\partial w}{\partial t}\right]_L \mathrm{d}t \neq 0.$$

Therefore, flutter is again predicted at infinitesimal flows because of the Coriolis forces, *à la* Païdoussis and Luu (1985), followed by restabilization at somewhat higher U than originally predicted (because of the $\frac{1}{2}$ factor).

The author believes that item (i) in Kuiper and Metrikine (2005) critique is correct, and item (ii) has some merit. Therefore, a reappraisal of the dynamics of the system was initiated, as described in greater detail in Païdoussis et al. (2005).

4.3. Reconsideration of the intake dynamics

Several conceptual models have been explored, some of which are presented in Païdoussis et al. (2005). Here, only one, the simplest, will be presented, and some of the results from the others discussed.

Fig. 16(a) shows the end of the pipe inclined at an angle $\chi \equiv \tan^{-1}(\partial w/\partial x)_L \simeq w'_L$, where $(\quad)' = \partial w/\partial x$. We postulate a small mean flow velocity $-v$ facing the intake, instead of taking $v = 0$ as in Païdoussis (1999). Moreover, we postulate that $-v$ remains in the *average* direction of the pipe end (presuming that oscillatory motion takes place), i.e., tangential

[12] In fact, Kuiper and Metrikine (2005) consider two limiting values for $\overline{p}$, namely $-\frac{1}{2}\rho U^2$ and $-\rho U^2$, the latter corresponding to that proposed in Païdoussis (1999).

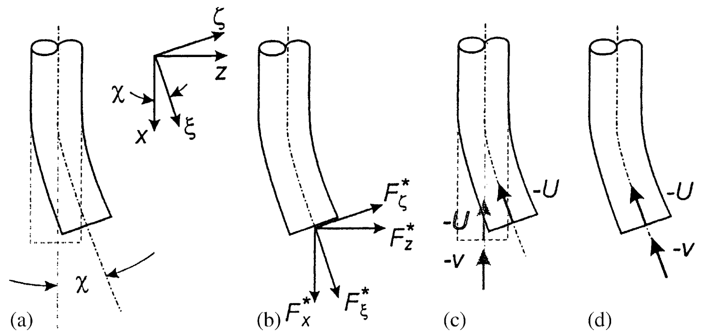

Fig. 16. (a), (b) Diagrams of the intake of the aspirating cantilever and definitions of some of the quantities used in the analysis, including the forces exerted by the fluid on the pipe at the inlet. (c) The mean flow velocity $-v$ facing the intake, presumed to remain in the mean (vertical) direction during a presumed cycle of oscillation, and the flow velocity in the pipe $-U$; (d) the same, presuming that the mean flow facing the intake is always tangential to the instantaneous configuration of the inlet (Païdoussis et al., 2005).

to the undeflected pipe as in Fig. 16(c). Hence, the forces exerted on the fluid at the inlet, equal to the change in momentum, $M\,U(\Delta U)$, are

$$F_x = MU[-U\cos\chi - (-v)] = -MU(U - v), \tag{14a}$$

$$F_z = MU[(\dot{w}_L - U\sin\chi) - 0] = MU(\dot{w}_L - U\,w'_L), \tag{14b}$$

correct to $\mathcal{O}(\varepsilon)$, where $w \sim \mathcal{O}(\varepsilon)$. In F_z, it is recognized that the fluid in the pipe has a velocity $\dot{w}_L$ in the z-direction, equal to the pipe-end velocity, as well as the tangential flow velocity $-U$; whereas, outside the pipe, the velocity in the z-direction is null. Most of these ideas were in fact proposed earlier by Pramila (1992), but unfortunately were not pursued.

Consequently, the forces of the fluid on the pipe, denoted by an asterisk, are

$$F^*_x = M\,U^2(1-\alpha), \quad F^*_z = -MU(\dot{w}_L - U\,w'_L), \tag{15}$$

where $\alpha = v/U$. It is of interest to compare this to previous work. According to the Païdoussis and Luu (1985) model, $\alpha = 1$; according to Païdoussis (1999), $\alpha = 0$; and Kuiper and Metrikine (2005) propose that α should be between $\alpha = \frac{1}{2}$ and $\alpha = 0$.

In order to obtain the end-shear force normal to the deformed pipe-end, and in accordance with a more precise derivation via Hamilton's principle, we express these forces in the (ξ, ζ) frame (Fig. 16(b)), and thus obtain

$$F^*_\xi \simeq F^*_x, \quad F^*_\zeta = F^*_y\cos\chi - F^*_x\sin\chi = -M\,U(\dot{w}_L - \alpha\,U\,w'_L). \tag{16}$$

The equation of motion continues to be Eq. (13), but since $F_\xi = M\,U^2(1-\alpha)$ this can be written as

$$EI\,\frac{\partial^4 w}{\partial x^4} + \alpha\,M\,U^2\frac{\partial^2 w}{\partial x^2} - 2MU\frac{\partial^2 w}{\partial x \partial t} + (M + m + M_a)\,\frac{\partial^2 w}{\partial t^2} = 0. \tag{17}$$

At the free end ($x = L$), the bending moment is zero, and the shear force is related to F^*_ζ; i.e.,

$$EI\,\frac{\partial^3 w}{\partial x^3} - MU\left(\frac{\partial w}{\partial t} - \alpha\,U\,\frac{\partial w}{\partial x}\right) = 0. \tag{18}$$

In this case, calculating the work done by the fluid on the pipe over a putative cycle of oscillation, one obtains

$$\Delta W = 0, \tag{19}$$

irrespective of what the value of α or how high U may be.

This "nice" conclusion depends crucially on presuming the inlet flow velocity to be as in Fig. 16(c). If, instead, the mean flow velocity vector facing the intake is presumed to always be tangential to the *deformed* pipe-end, as in Fig. 16(d), then ΔW is generally not zero — although it can be zero, depending on the precise assumptions made for the flow around the inlet. It is also noted that in the foregoing $\overline{T} = T_L$ was presumed to be zero, though it can be substantial. More importantly, if oscillation occurs, one should use the *unsteady* Bernoulli equation to calculate $\overline{p} = p_L$. Both of these would increase $(\overline{T} - \overline{p}A)$ in Eq. (13), tending to cancel the centrifugal force and hence to stabilize the system. In this case, it is found (Païdoussis et al. (2005)) that

$$\Delta W = -[1-(1-\alpha)(1+\overline{\gamma})]M\,U^2 \int_0^T \left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial t}\right)\bigg|_L \mathrm{d}t, \tag{20}$$

where α is as in Eqs. (15) and $\overline{\gamma}$ is a parameter related to the suction (tension) on the pipe-face itself at the free end. Since, $\alpha \sim \mathcal{O}(\frac{1}{2})$ though less than 0.5 and $\overline{\gamma} \sim \mathcal{O}(1)$, it is seen that generally ΔW is small. Hence, in cases where $\Delta W \neq 0$, and hence an instability is possible, the rate of energy input is small. Therefore, for the cases where instability is predicted, if a realistic amount of dissipation, neglected so far, is taken into account, the system is found to remain stable up to flow velocities covering the range of practical interest (for the ocean mining application, for example).

It is important to stress here that, whether the flow is as for Fig. 16(c) or (d), the Coriolis term in the equations of motion is cancelled out by the term $-M\,U(\partial w/\partial t)$ in the boundary condition, insofar as the calculation of ΔW is concerned. Flutter, if it occurs, is related to a nonvanishing fraction of $M\,U^2(\partial^2 w/\partial x^2)$, the centrifugal force.

4.4. Concluding comment

From the fundamentals perspective, the question remains not wholly resolved, since, depending on the precise assumptions made on the intake flow structure, flutter *can* be predicted at infinitesimal flow if dissipative forces are ignored.[13] Therefore, in the experiment described in the penultimate paragraph of Section 4.1, was the observed stability only due to dissipation? Or was it because the flow at the intake is such as to make flutter impossible? To help get to the bottom of things, a CFD study of the flow field has been initiated.

5. Conclusion

Three unresolved, or more precisely partly resolved, problems involving fluid-structure interaction have been discussed in this paper — three among many in the field.

All three proved to be only deceptively simple. Therefore, the path towards their resolution has been tortuous, marked by revisions and reversals in the explanations proposed and presumed solutions of the ensuing paradoxes and the quandaries posed thereby.

On the question of existence/nonexistence of post-divergence flutter of shells with supported ends and conveying fluid, the problem in which most progress has been achieved, invaluable input was supplied by experiment. In the case of existence/nonexistence of infinitesimal-flow flutter in fluid-conveying shells with mismatched end-supports, however, experimental corroboration is difficult, for the reasons already remarked. Similarly, definitive experimental proof of the existence/nonexistence of infinitesimal-flow flutter of aspirating pipes would be difficult — though, with enough ingenuity, not impossible.

It is hoped that this paper has contributed, as many others emanating from the FIV-2004 Conference in Paris have (to be found in some other 2005 issues of JFS), towards showing how intricate, exciting and full of surprises the study of Fluid-Structure Interactions can be.

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[13] In terms of applications, if dissipation is accounted for, instabilities generally occur at impractically high flows [cf. Xia et al. (2004), Deepak et al. (2001)].

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